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Regarding the axial-vector mesons

D.-M. Li[1,2,a], B. Ma[1], and H. Yu[2]

[1] Department of Physics, Zhengzhou University, Zhengzhou, Henan 450052, PRC[b]
[2] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, PRC

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Abstract. The implications of the $f_1(1285)$-$f_1(1420)$ mixing for the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle are investigated. Based on the $f_1(1285)$-$f_1(1420)$ mixing angle $\sim 50°$ suggested from the analysis for a substantial body of data concerning the $f_1(1420)$ and $f_1(1285)$, the masses of the $K_1(^3P_1)$ and $K_1(^1P_1)$ are determined to be $\sim 1307.35 \pm 0.63$ MeV and 1370.03 ± 9.69 MeV, respectively, which therefore suggests that the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle is about $\pm(59.55 \pm 2.81)°$. Also, it is found that the mass of the $h_1'(^1P_1)$ (mostly of $s\bar{s}$) state is about 1495.18 ± 8.82 MeV. Comparison of the predicted results and the available experimental information of the $h_1(1380)$ shows that without further confirmation on the $h_1(1380)$, the assignment of the $h_1(1380)$ as the $s\bar{s}$ member of the $^1P_1$ meson nonet may be premature.

PACS. 14.40.Ev Other strange mesons – 12.40.Yx Hadron mass models and calculations

1 Introduction

The strange axial-vector mesons provide interesting possibilities to study the QCD in the nonperturbative regime by the mixing of the $^3P_1$ and $^1P_1$ states. In the exact $SU(3)$ limit, the $K_1(^3P_1)$ and $K_1(^1P_1)$ do not mix, just as the a_1 and b_1 mesons do not mix. For the strange quark mass greater than the up- and down-quark masses so that $SU(3)$ is broken, also the $K_1(^3P_1)$ and $K_1(^1P_1)$ do not possess definite C-parity; therefore these states can in principle mix to give the physical $K_1(1270)$ and $K_1(1400)$.

In the literature, the mixing angle of the $K_1(^3P_1)$ and $K_1(^1P_1)$, θ_K has been estimated by some different approaches; however, there is not yet a consensus on the value of θ_K. As the optimum fit to the data as of 1977, Carnegie *et al.* find $\theta_K = (41 \pm 4)°$ [1]. Within the heavy-quark effective theory Isgur and Wise predict two possible mixing angles, $\theta_K \sim 35.3°$ and $\theta_K \sim -54.7°$ [2]. Based on the analysis of $\tau \to \nu K_1(1270))$ and $\tau \to \nu K_1(1400))$, Rosner suggests $\theta_K \sim 62°$ [3], Asner *et al.* give $\theta_K = (69 \pm 16 \pm 19)°$ or $(49 \pm 16 \pm 19)°$ [4], and Cheng obtain $\theta_K = \pm 37°$ or $\pm 58°$ [5]. From the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki finds two possible solutions with a twofold ambiguity, $\theta_K \sim 33°$ or $57°$ [6]. A constraint $35° \leq \theta_K \leq 55°$ is predicted by Burakovsky *et al.* in a nonrelativistic constituent-quark model [7], and within the same model, the values of $\theta_K \simeq (31 \pm 4)°$ and $\theta_K \simeq (37.3 \pm 3.2)°$ are suggested by Chliapnikov [8] and Burakovsky [9], respectively. The calculations for the strong decays of $K_1(1270)$ and $K_1(1400)$ in the $^3P_0$ decay model suggest $\theta_K \sim 45°$ [10,11]. The mixing angles $\theta_K \sim 34°$ [12], $\theta_K \sim 5°$ [13] are also presented within a relativized quark model. More recently, Vijande *et al.* suggest $\theta_K \sim 55.7°$ based on the calculations in a constituent-quark model [14].

It is widely believed that the $f_1(1285)$ and $f_1(1420)$ are the isoscalar states of the $^3P_1$ meson nonet [15]. The analysis of the Gell-Mann–Okubo mass formula, $SU(3)$ coupling formula, radiative decay of the $f_1(1285)$, $\gamma\gamma^*$ decays of the $f_1(1285)$ and $f_1(1420)$, and the radiative J/ψ decays performed by Close and Kirk [16], indicates that these various data are independently consistent with the $f_1(1285)$-$f_1(1420)$ mixing angle $\alpha \sim 50°$ (in the singlet-octet basis). This value of $\alpha \sim 50°$ is also supported by the calculations performed by [14,17–19].

We shall show below that the mass of the $K_1(^3P_1)$ can be related to the mass matrix describing the mixing of the $f_1(1285)$ and $f_1(1420)$, and the $f_1(1285)$-$f_1(1420)$ mixing angle can give a constraint on the mixing $K_1(^3P_1)$-$K_1(^1P_1)$. The main purpose of the present work is to discuss the implications of the $f_1(1285)$-$f_1(1420)$ mixing for the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle.

2 The mixing angle of $K_1(^3P_1)$ and $K_1(^1P_1)$

In the $N = (u\bar{u} + d\bar{d})/\sqrt{2}$, $S = s\bar{s}$ basis, the mass-squared matrix describing the mixing of the $f_1(1420)$ and $f_1(1285)$

[a] e-mail: lidm@zzu.edu.cn
[b] Mailing address.

can be written as [20]

$$M^2 = \begin{pmatrix} M^2_{a_1(^3P_1)} + 2\beta & \sqrt{2}\beta X \\ \sqrt{2}\beta X & 2M^2_{K_1(^3P_1)} - M^2_{a_1(^3P_1)} + \beta X^2 \end{pmatrix}, \quad (1)$$

where $M_{a_1(^3P_1)}$ and $M_{K_1(^3P_1)}$ are the masses of the states $a_1(^3P_1)$ and $K_1(^3P_1)$, respectively; β denotes the total annihilation strength of the $q\bar{q}$-pair for the light flavors u and d; X describes the $SU(3)$-breaking ratio of the nonstrange and strange quark propagators via the constituent-quark mass ratio m_u/m_s. The masses of the two physical isoscalar states $f_1(1420)$ and $f_1(1285)$, M_1 and M_2, can be related to the matrix M^2 by the unitary matrix U,

$$M^2 = U^\dagger \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix} U, \quad (2)$$

and the physical states $f_1(1420)$ and $f_1(1285)$ can be expressed as

$$\begin{pmatrix} f_1(1420) \\ f_1(1285) \end{pmatrix} = U \begin{pmatrix} N \\ S \end{pmatrix}. \quad (3)$$

Also, in the basis $\mathbf{8} = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, $\mathbf{1} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, the mixing of the $f_1(1420)$ and $f_1(1285)$ can be expressed by

$$\begin{pmatrix} f_1(1420) \\ f_1(1285) \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathbf{8} \\ \mathbf{1} \end{pmatrix}, \quad (4)$$

where α is the $f_1(420)$-$f_1(1285)$ mixing angle in the singlet-octet basis.

With the help of

$$\begin{pmatrix} \mathbf{8} \\ \mathbf{1} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}, \quad (5)$$

from (3) and (4), one can have

$$U = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \quad (6)$$

Based on (1), (2) and (6), the following relations can be obtained

$$M^2_{a_1(^3P_1)} + 2\beta = \left(\sqrt{\frac{1}{3}}\cos\alpha - \sqrt{\frac{2}{3}}\sin\alpha\right)^2 M_1^2 + \left(\sqrt{\frac{2}{3}}\cos\alpha + \sqrt{\frac{1}{3}}\sin\alpha\right)^2 M_2^2, \quad (7)$$

$$\sqrt{2}\beta X = \left(\sqrt{\frac{1}{3}}\cos\alpha - \sqrt{\frac{2}{3}}\sin\alpha\right) \times \left(\sqrt{\frac{2}{3}}\cos\alpha + \sqrt{\frac{1}{3}}\sin\alpha\right)(M_2^2 - M_1^2), \quad (8)$$

$$2M^2_{K_1(^3P_1)} - M^2_{a_1(^3P_1)} + \beta X^2 = \left(\sqrt{\frac{1}{3}}\cos\alpha - \sqrt{\frac{2}{3}}\sin\alpha\right)^2 M_2^2 + \left(\sqrt{\frac{2}{3}}\cos\alpha + \sqrt{\frac{1}{3}}\sin\alpha\right)^2 M_1^2. \quad (9)$$

The constituent-quark mass ratio can be determined within the nonrelativistic constituent-quark model (NRCQM). In the NRCQM [8,9], the mass of a $q\bar{q}$ state with $L = 0$, $M_{q\bar{q}}$, is given by

$$M_{q\bar{q}} = m_q + m_{\bar{q}} + \Lambda \frac{\mathbf{s}_q \cdot \mathbf{s}_{\bar{q}}}{m_q m_{\bar{q}}}, \quad (10)$$

where m and $\mathbf{s}$ are the constituent-quark mass and spin, Λ is a constant. Since $\mathbf{s}_q \cdot \mathbf{s}_{\bar{q}} = -3/4$ for spin-0 mesons and $1/4$ for spin-1 mesons, in the $SU(2)$ flavor symmetry limit, one can have

$$X \equiv \frac{m_u}{m_s} = \frac{M_\pi + 3M_\rho}{2M_K + 6M_{K^*} - M_\pi - 3M_\rho} = 0.6298 \pm 0.00068. \quad (11)$$

Taking $\alpha \simeq 50°$ obtained from several independent analyses [16] as mentioned in sect. 1, $M_1 = 1426.3 \pm 0.9$ MeV and $M_2 = 1281.8 \pm 0.6$ MeV [15], from relations (7)–(9), we have[1]

$$M_{K_1(^3P_1)} \simeq 1307.35 \pm 0.63 \text{ MeV}, \quad M_{a_1(^3P_1)} \simeq 1205.06 \pm 0.92 \text{ MeV}. \quad (12)$$

The $K_1(^3P_1)$ and $K_1(^1P_1)$ can mix to produce the physical states $K_1(1400)$ and $K_1(1270)$ and the mixing between $K_1(^3P_1)$ and $K_1(^1P_1)$ can be parameterized as [6]

$$K_1(1400) = K_1(^3P_1)\cos\theta_K - K_1(^1P_1)\sin\theta_K, \quad K_1(1270) = K_1(^3P_1)\sin\theta_K + K_1(^1P_1)\cos\theta_K, \quad (13)$$

where θ_K denotes the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle. Without any assumption about the origin of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing, the masses of the $K_1(^3P_1)$ and $K_1(^1P_1)$ can be related to $M_{K_1(1400)}$ and $M_{K_1(1270)}$, the masses of the $K_1(1400)$ and $K_1(1270)$, by the following relation phenomenologically,

$$S \begin{pmatrix} M^2_{K_1(^3P_1)} & A \\ A & M^2_{K_1(^1P_1)} \end{pmatrix} S^\dagger = \begin{pmatrix} M^2_{K_1(1400)} & 0 \\ 0 & M^2_{K_1(1270)} \end{pmatrix}, \quad (14)$$

where A denotes a parameter describing the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing, and

$$S = \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}.$$

From (14), one can have

$$M^2_{K_1(^3P_1)} = M^2_{K_1(1400)}\cos^2\theta_K + M^2_{K_1(1270)}\sin^2\theta_K, \quad (15)$$

$$M^2_{K_1(^1P_1)} = M^2_{K_1(1400)}\sin^2\theta_K + M^2_{K_1(1270)}\cos^2\theta_K, \quad (16)$$

$$\cos(2\theta_K) = \frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}}. \quad (17)$$

[1] Here $\beta \simeq 108078.0 \pm 834.788$ MeV^2.

Inputting $M_{K_1(1400)} = 1402 \pm 7$ MeV, $M_{K_1(1270)} = 1273 \pm 7$ MeV [15] and $M_{K_1(^3P_1)} \simeq 1307.35 \pm 0.63$ MeV shown in (12), from (15)–(17), we have

$$\begin{aligned} M_{K_1(^1P_1)} &\simeq 1370.03 \pm 9.69 \text{ MeV}, \\ |\theta_K| &\simeq (59.55 \pm 2.81)^\circ . \end{aligned} \tag{18}$$

Recently, based on the relations (15)–(17) and restricting to $0 < \theta_K < 90°$, Nardulli and Pham found [21]

$$\begin{aligned} &[\text{solution a}]: (M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = \\ &\qquad (1310, 1367) \text{ MeV, for } \theta_K = 32^\circ, \\ &[\text{solution b}]: (M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = \\ &\qquad (1367, 1310) \text{ MeV, for } \theta_K = 58^\circ . \end{aligned}$$

Our predicted result that $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) \simeq (1370, 1307)$ MeV and $|\theta_K| \simeq 59.55°$ extracted from $\alpha \simeq 50°$ is in excellent agreement with the solution b given by [21].

Within the nonrelativistic constituent-quark model, the results regarding the masses of the $K_1(^1P_1)$ and $K_1(^3P_1)$, $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1368, 1306)$ MeV suggested by [8] and $(M_{K_1(^1P_1)}, M_{K_1(^3P_1)}) = (1356, 1322)$ MeV suggested by [9], are in good agreement with our predicted result. However, based on the following relation employed by [8, 9]:

$$\tan^2(2\theta_K) = \left(\frac{M^2_{K_1(^3P_1)} - M^2_{K_1(^1P_1)}}{M^2_{K_1(1400)} - M^2_{K_1(1270)}} \right)^2 - 1, \tag{19}$$

the values of $\theta_K = (31 \pm 4)°$ given by [8] and $\theta_K = (37.3 \pm 3.2)°$ given by [9] disagree with the value of $|\theta_K| \simeq (59.55 \pm 2.81)°$ given by the present work.

Obviously, (19) is equivalent to (17), and will yield two solutions $|\theta_K|$ and $\frac{\pi}{2} - |\theta_K|$. Simultaneously, considering the relations (15), (16) and (19), in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, we can conclude that if $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$, the $|\theta_K|$ would be greater than 45°. In fact, relation (17) clearly indicates that in the presence of $M_{K_1(1400)} > M_{K_1(1270)}$, the case $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$ must require $45° < |\theta_K| < 90°$.

In the framework of a covariant light-front quark model, the calculations performed by Cheng and Chua [22] for the exclusive radiative B decays, $B \to K_1(1270)\gamma$, $K_1(1400)\gamma$, show that the relative strength of $B \to K_1(1270)\gamma$ and $B \to K_1(1270)\gamma$ rates is very sensitive to the sign of the $K_1(1270)$-$K_1(1400)$ mixing angle. For $\theta_K = \pm 58°$, the following relation is predicted [22]:

$$\frac{\mathcal{B}(B \to K_1(1270)\gamma)}{\mathcal{B}(B \to K_1(1270)\gamma)} = \begin{cases} 10.1 \pm 6.2 & \text{for } \theta_K = +58^\circ, \\ 0.02 \pm 0.02 & \text{for } \theta_K = -58^\circ. \end{cases} \tag{20}$$

Evidently, the experimental measurement of the above ratio of branching fractions can be used to fix the sign of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle. Recently, the first measurement of the branching ratio $\mathcal{B}$ for B decay into $K_1(1270)\gamma$, together with an upper bound on $K_1(1400)$, $\mathcal{B}(B^+ \to K_1^+(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$, $\mathcal{B}(B^+ \to K_1^+(1400)\gamma) < 1.44 \times 10^{-5}$ has been reported by the Belle Collaboration [23]. Based on the measurements of the Belle Collaboration [23], the analysis of the radiative B decays with an axial-vector meson in the final state performed by Nardulli and Pham [21] within naive factorization suggests that $\mathcal{B}(B^+ \to K_1^+(1400)\gamma) = 4.4 \times 10^{-6}$ for $\theta_K = +58°$, which is consistent with the predictions given by [22]. Further experimental studies of $\mathcal{B}(B^+ \to K_1^+(1270)\gamma)$ and $\mathcal{B}(B^+ \to K_1^+(1400)\gamma)$ is certainly desirable for understanding the sign of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle.

Our predicted center value of the $a_1(^3P_1)$ mass is ~ 1205.06 MeV, slightly smaller than the measured center value of the $a_1(1260)$ mass, 1230 MeV, although the predicted value 1205.06 ± 0.92 MeV is consistent with the experimental datum 1230 ± 40 MeV within errors. A similar result has been obtained by Chliapnikov within the NRCQM [8]. According to the NRCQM prediction that if $M_{K_1(^3P_1)} < M_{K_1(^1P_1)}$, $M_{a_1(^3P_1)}$ would be less than $M_{b_1(^1P_1)}$ [8, 9], therefore, in the presence of $M_{K_1(^3P_1)} \simeq 1307 < M_{K_1(^1P_1)} \simeq 1370$ MeV, the $a_1(^3P_1)$ mass should smaller than the $b_1(1230)$ mass (1229.5 ± 3.2 MeV [15]). In addition, notice that the determination of the $a_1(1260)$ mass in hadronic production and in $\tau \to a_1\nu_\tau$ decay is to a certain extent model dependent [15].

3 The $s\bar{s}$ member of the $^1P_1$ meson nonet

According to PDG [15], the $h_1(1170)$ as the $^1P_1$ isoscalar state (mostly of $u\bar{u}+d\bar{d}$) is well established experimentally. However, the assignment of the $s\bar{s}$ partner of the $h_1(1170)$ remains ambiguous. In the presence of the $b_1(1235)$ and $h_1(1170)$ being the members of the $^1P_1$ meson nonet, with the help of the $K_1(^1P_1)$ mass obtained in sect. 2, we shall estimate the mass of the $^1P_1$ $s\bar{s}$ state using different approaches.

By applying (1) and (2) to the $^1P_1$ meson nonet, we can obtain the following relations:

$$\begin{aligned} &2M^2_{K_1(^1P_1)} + (2 + X^2)\beta_1 = M^2_{h_1(1170)} + M^2_{h_1'}, \\ &(M^2_{b_1(1235)} + 2\beta_1)(2M^2_{K_1(^1P_1)} \\ &-M^2_{b_1(1235)} + \beta_1 X^2) - 2\beta_1^2 X^2 = M^2_{h_1(1170)} M^2_{h_1'}, \end{aligned} \tag{21}$$

where h_1' denotes the $s\bar{s}$ partner of the $^1P_1$ states $h_1(1170)$ and $b_1(1235)$. Using $M_{K_1(^1P_1)} \simeq 1370.03 \pm 9.69$ MeV, $X = 0.6298 \pm 0.00068$ obtained in sect. 2, and the measured values $M_{b_1(1235)} = 1229.5 \pm 3.2$ MeV and $M_{h_1(1170)} = 1170 \pm 20$ MeV [15], we have

$$\begin{aligned} \beta_1 &\simeq -(69143.5 \pm 22373.6) \text{ MeV}^2, \\ M_{h_1'} &\simeq 1489.75 \pm 18.08 \text{ MeV}. \end{aligned} \tag{22}$$

Then from (1) and (2), the quarkonia content of the $h_1(1170)$ and $h_1'(1490)$ can be given by

$$\begin{pmatrix} h_1'(1490) \\ h_1(1170) \end{pmatrix} \simeq \begin{pmatrix} 0.073 \pm 0.02 & -(0.997 \pm 0.002) \\ 0.997 \pm 0.002 & 0.073 \pm 0.02 \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}. \tag{23}$$

Equations (22) and (23) indicate that with the $b_1(1230)$, $h_1(1170)$ and $K_1(1370)$ in the $^1P_1$ meson nonet, another isoscalar state of the $^1P_1$ meson nonet, h'_1, would have a mass of about 1490 MeV and is composed mostly of $s\bar{s}$.

Considering the fact that the $f'_2(1525)$ is an almost pure $s\bar{s}$ state [20], we obtain the estimated mass of the $^1P_1$ $s\bar{s}$ state from the following relation given by the NR-CQM [8]:

$$M_{s\bar{s}(^1P_1)} = M_{f'_2(1525)} - (M_{a_2(1320)} - M_{b_1(1235)})X^2 = 1489.78 \pm 5.16 \text{ MeV}, \tag{24}$$

which is in excellent agreement with $M_{h'_1} \simeq 1489.75 \pm 18.08$ MeV shown in (22).

Also, in the framework of the quasi-linear Regge trajectory (see ref. [19] and references therein), *i.e.*,

$$J = \alpha_{i\bar{i}'}(0) + \alpha'_{i\bar{i}'} M^2_{i\bar{i}'}, \tag{25}$$

where i ($\bar{i}'$) refers to the quark (antiquark) flavor, J and $M_{i\bar{i}'}$ are, respectively, the spin and mass of the $i\bar{i}'$ meson, $\alpha_{i\bar{i}'}(0)$ and $\alpha'_{i\bar{i}'}$ are, respectively, the intercept and slope of the trajectory on which the $i\bar{i}'$ meson lies; For a meson multiplet, the parameters for different flavors can be connected by the following relations:

i) additivity of intercepts,

$$\alpha_{i\bar{i}}(0) + \alpha_{j\bar{j}}(0) = 2\alpha_{j\bar{i}}(0); \tag{26}$$

ii) additivity of inverse slopes,

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{j\bar{i}}}; \tag{27}$$

for the $^1P_1$ $q\bar{q}$ nonet, one can have[2]

$$M_{s\bar{s}(^1P_1)} = \left[\frac{2\alpha'_{n\bar{s}} M^2_{K_1(^1P_1)} - \alpha'_{n\bar{n}} M^2_{b_1(1235)}}{\alpha'_{s\bar{s}}}\right]^{\frac{1}{2}} = 1506.01 \pm 18.62 \text{ MeV}, \tag{28}$$

which is also consistent with $M_{h'_1} \simeq 1489.75 \pm 18.08$ MeV given in (22).

In the presence of the $b_1(1235)$, $h_1(1170)$ and $K_1(^1P_1)$ (with a mass of about 1370 MeV) belonging to the $^1P_1$ meson nonet, the above three different and complementary approaches, *i.e.*, meson-meson mixing, nonrelativistic constituent-quark model and Regge phenomenology, consistently suggest that the ninth member of the $^1P_1$ nonet has a mass of about 1495.18 ± 8.82 MeV (averaged value of the above three predicted results) and is mainly strange. Our predicted mass of the $^1P_1$ $s\bar{s}$ state is in good agreement with the values 1499 ± 16 MeV suggested by Chliapnikov in a nonrelativistic constituent-quark model [8] and 1511 MeV recently found by Vijande *et al.* in a constituent-quark model [14].

[2] Here we take $\alpha'_{n\bar{n}} = 0.7218$, $\alpha'_{s\bar{s}} = 0.6613$ and $\alpha'_{n\bar{s}} = 0.6902$ GeV^{-2} [19].

Experimentally, the $h_1(1380)$ with $J^{PC} = 1^{+-}$ was claimed to be observed in the $K\overline{K}\pi$ system by only two collaborations, the LASS Collaboration [24] (mass: 1380 ± 20 MeV, $\Gamma = 80 \pm 30$ MeV) and the Crystal Barrel Collaboration [25] (mass: 1440 ± 60 MeV, $\Gamma = 170 \pm 80$ MeV), and the observed decay mode of the $h_1(1380)$ ($K\overline{K}^*$) favors the assignment of the $h_1(1380)$ as a $s\bar{s}$ state.

On the one hand, our predicted mass of the $^1P_1$ $s\bar{s}$ state, 1495.18 ± 8.82 MeV, is significantly larger than 1380 ± 20 MeV. The prediction given by Godfrey and Isgur in a relativized quark model [12] for the mass of the $^1P_1$ $s\bar{s}$ state is 1.47 GeV, at least 70 MeV higher than the measured result of LASS [24]. Therefore, if the measured results of LASS [24] were confirmed, the $h_1(1380)$ would seem too light to be the $^1P_1$ $s\bar{s}$ member. The studies on the implications of large N_c and chiral symmetry for the mass spectra of meson resonances performed by Cirigliano *et al.* [26] also disfavor the assignment of the $h_1(1380)$ to $^1P_1$ $s\bar{s}$.

On the other hand, the predicted mass of the $^1P_1$ $s\bar{s}$ state is consistent with 1440 ± 60 MeV within errors, and the calculations performed by Barnes *et al.* [11] for the total width of the $^1P_1$ $s\bar{s}$ state in the $^3P_0$ decay model also show that at this mass the assignment of the $h_1(1380)$ as the $^1P_1$ $s\bar{s}$ state appears plausible. So, if the measured results of Crystal Barrel [25] were confirmed, the $h_1(1380)$ would be a convincing candidate for the $s\bar{s}$ partner of the $^1P_1$ state $h_1(1170)$.

Notice that the uncertainties of these measurements are rather large, and the $h_1(1380)$ state still needs further confirmation [15]. Without confirmed experimental information about the $h_1(1380)$, the present results indicate that the assignment of the $h_1(1380)$ as the $^1P_1$ $s\bar{s}$ member may be premature.

4 Concluding remarks

The studies on the implications of the $f_1(1285)$-$f_1(1420)$ mixing for the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle indicate that the $f_1(1285)$-$f_1(1420)$ mixing angle $\sim 50°$ suggested by Close *et al.* [16] implies that $(M_{K_1(^3P_1)}, M_{K_1(^1P_1)}) \simeq (1307, 1370)$ MeV, which therefore suggests that the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle $\simeq \pm 59.55°$. The experimental measurement of the ratio of $B \to K_1(1270)\gamma$ and $B \to K_1(1270)\gamma$ rates can be used to fix the sign of the $K_1(^3P_1)$-$K_1(^1P_1)$ mixing angle. Also, with the $b_1(1235)$, $h_1(1170)$ and $K_1(^1P_1)$ in the $^1P_1$ meson nonet, three different and complementary approaches, *i.e.*, meson-meson mixing, nonrelativistic constituent-quark model and Regge phenomenology, consistently suggest that the $^1P_1$ $s\bar{s}$ member has a mass of about 1495.18 MeV. Our predicted mass of the $^1P_1$ $s\bar{s}$ state is significantly larger than the measured value of the $h_1(1380)$ mass reported by LASS [24], while it is consistent with that reported by Crystal Barrel [25], which shows that without further confirmation on the $h_1(1380)$, the assignment of the $h_1(1380)$ remains open.

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